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MARKET SEARCH MODELS: A SELECTIVE SURVEY

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### I. INTRODUCTION

The economics of information has become quite a growth industry in the last few years, so this survey will be very specific, and somewhat arbitrary. I am going to focus on a subset of models belonging to that area of research currently being referred to as the economics of imperfect information. The models in this class all relax, in one way or another, the assumption that each agent in the economy has perfect information regarding all market parameters. Uncertain parameters might include risk characteristics of consumers, worker or product quality, propensities of workers for quitting, or the specific wages (or prices) offered by different firms. The last imperfection in this representative list was initially investigated by George Stigler in a couple of classic papers published in the Journal of Political Economy in the early sixties. The papers I will discuss here all follow in the tradition sparked by Stigler, which has come to be known as search theory. The methodology can be applied to any market and, indeed, a large portion of the literature is set in product markets. However, I will confine my discussion and examples to the labor market when possible.

The standard neoclassical economic model completely ignores information. It does this by assuming that all agents know everything that is going on in the economy. But this is a very strong assumption and certainly a good deal of observable phenomena is related to the fact that agents do not know everything about the markets in which they participate. One fundamental

aspect of any market is the price or wage at which transactions take place. If there are no transactions costs then some kind of barter process will presumably lead to a single wage in equilibrium. Implicit in the assumption of no transactions costs is the concept of perfect information; by introducing imperfect information the assumption of no transactions costs is changed also. The key is specifying the nature of the transactions costs. Search theory does this in a very unique way: workers are assumed to not know where to find particular wage offers; they may know the general distribution of wage offers, but they have to incur some "search cost" to discover what wage any given firm is paying. Thus, as a worker looks for wage offers he may stop short of the maximum wage available in the market -- with positive search costs it just doesn't pay to hold out for the highest conceivable wage in all cases. There is clearly a problem here for the searching worker; he must choose a search rule (how to go about gathering offers) and he must determine the optimal use of this rule. Early work on search theory focused on this problem, and I will discuss two seminal papers which analyzed the worker's problem, Stigler [1962] and McCall [1970].

But, as the title of this survey suggests, I aim to focus on market models of search. Stigler and McCall analyze only one side of the market at a time. In particular, they take as given the dispersion of wage offers which a worker faces, and with no dispersion there is no search problem. Market search models introduce the firm side explicitly and attempt to define conditions under which a stable, non-degenerate dispersion of wages will arise. Section II discusses Stigler and McCall as a further introduction and background to these market models. Sections III and IV present the market models themselves.<sup>1</sup>

The following criteria are relevant to the market models I will discuss: symmetry, placement of the search burden, and sequentiality. Symmetry refers to the treatment of vacancies in relation to the treatment of unemployed workers. That is, there are several ways one could introduce the firm side of the market. One way is to treat firms as consisting of some given number of jobs. When a job vacancy occurs, it is treated by the firm in a way exactly analogous to the way an unemployed worker is treated; some search rule for locating potential employees is selected and an optimal strategy defined. Asymmetric models treat vacant jobs as essentially different creatures than unemployed workers. Profit constraints are imposed and, generally speaking, one side of the market is given the burden of seeking out members of the other side. This is the second criterion mentioned above; who bears the burden of search. Of course it is relevant only to the asymmetric models since by definition the burden is shared in the symmetric models. The final consideration is the type of search strategy. Section II introduces two; optimal sample size (Stigler) and sequential (McCall). The former says to first decide how many firms to sample and then pick the one offering the highest wage. The latter says to sample firms one at a time until a satisfactory wage is found. Other strategies are possible and I class models broadly according to sequential or non-sequential.

Thus, section III and IV break down in the following way. Section III focuses on sequential market models. Mortensen [1973] is a symmetric sequential model and Wilde [1975] is an asymmetric sequential model with the burden of search falling on the worker. Section IV focuses on non-sequential asymmetric models. Butters [1975] places the burden of search on the firm and Wilde [1976] puts it on the worker. The classification is summarized in the following

table:

- I. Sequential
  - 1. Symmetric [Mortensen, 1973]
  - 2. Asymmetric [Wilde, 1975]
- II. Non-sequential (asymmetric)
  - 1. Burden of search on firm [Butters, 1975]
  - 2. Burden of search on worker [Wilde, 1976]

Finally, section V will summarize and try to suggest some possibilities for future research.

## II. STIGLER AND McCALL

Stigler addressed the issue of why labor markets tend to maintain a "pure" dispersion of wages (one not associated with the heterogeneity of the work force or non-wage differences in jobs), and proposed that it is due to a lack of information on the part of workers and firms. That is, only if workers and firms have complete knowledge of all wage offers and reservation wages will a single wage emerge in equilibrium. Complete knowledge means that a worker knows precisely what wage can be obtained at every firm, and that each firm knows precisely what each potential employee's reservation wage<sup>2</sup> is. When this is not the case the market is characterized by uncertainty and costly information. The formal problem is to provide a mathematical representation of the process of information acquisition which will allow one to measure the benefits of search and weigh them against the costs. This can be a problem due to the obviously nebulous nature of information.

Stigler modeled the process in the following way. Suppose that a worker has some notion of the wage offers confronting him. Represent the offers by a probability density function, say  $\phi(w)$ .

The worker doesn't know where to find a particular offer, but he can invest some resources and get an observation. That is,  $c$  is the cost of drawing an observation at random from the distribution of wage offers,  $\phi(w)$ . This is how the worker gathers information in this market, where information consists of specific wage offers. The worker's problem, as Stigler formulated it, is to pick the optimal number of firms to sample. Clearly, if this optimal number is greater than one, the worker will work for the firm paying the highest wage. Thus, in determining the optimal sample size (weighing off the benefits of larger samples against the increased cost) the relevant distribution is not  $\phi(w)$ , but rather the distribution of the order statistic  $x_n = \max\{w_n\}$  which reflects the best offer given  $n$  draws from  $\phi(w)$ . The cumulative density function associated with  $x_n$  is given by  $G_n(w) = \text{Prob}\{x_n \leq w\} = \Phi(w)^n$ , where  $\Phi(w)$  is the cumulative density function of wage offers. Given  $G_n(w)$ , the expected best offer when  $n$  draws are taken is

$$(1) \quad E(x_n) = \int_{\underline{w}}^{\overline{w}} w g_n(w) dw,$$

where  $[\underline{w}, \overline{w}]$  defines the limits of the distribution of wage offers and  $g_n(w) = G'_n(w)$ . The expected gain from drawing  $n+1$  observations instead of  $n$  is just

$$(2) \quad H(n) = E(x_{n+1}) - E(x_n).$$

The optimal sample size,  $n^*$ , is given by setting the marginal expected gain from increasing the sample size by one against the marginal cost,  $c$ . Since  $H(n)$  is defined over discrete points, this condition reduces to

$$(3) \quad H(n^*) = E(x_{n^*+1}) - E(x_{n^*}) \leq c \leq E(x_{n^*}) - E(x_{n^*-1}) = H(n^* - 1).$$

Figure 1 illustrates this condition. It pays to take the  $n^*$ th sample, but not the  $(n^* + 1)$ th.

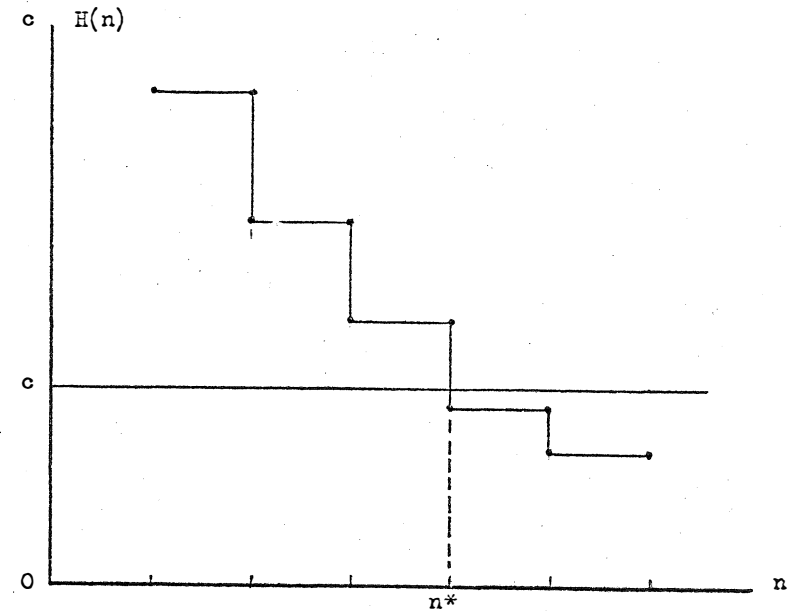


Figure 1: Optimal Sample Size

Subsequent analysis suggested that under the conditions defined by Stigler, the optimal policy would not necessarily be to take a fixed sample, but rather to search sequentially. McCall [1970] showed, in fact, that the following strategy is optimal when the constant cost of sampling is  $c$  per observation and the distribution  $\phi(w)$  is fixed and known. The worker should set a reservation wage  $w_r$  such that he searches sequentially until he finds a wage offer which beats  $w_r$ . That is, observations are taken one at a time and each is compared to some hypothetical wage level,  $w_r$ , defined as that wage at which the worker is indifferent to accepting the wage or paying the search cost  $c$  and drawing one more sample. Ignoring a number of complicating factors and assuming the worker seeks to maximize income net of search costs,  $w_r$  is defined formally by the following equation:

$$(4) \quad c = \int_{w_r}^{\bar{w}} (w - w_r) \phi(w) dw = J(w_r),$$

where as before  $\phi(w)$  is defined over  $[\underline{w}, \bar{w}]$ . The right hand side of (4) is the expected gain from one more sample given a current observation of  $w_r$ . In non-mathematical terms,  $J(w_r)$  weights wages above  $w_r$  by the probability of observing them in one draw and then adds up the gains  $(w - w_r)$ . Notice that no weight is put on offers below  $w_r$ . This total expected gain from one more draw is set against the marginal cost of the draw,  $c$ . This condition is illustrated in figure 2.

Intuitively, the reason why a sequential strategy is superior to the optimal sample size strategy used by Stigler is simple; under the sequential strategy the worker never overinvests in information. Suppose, for example, that you were using optimal sample size rules and you drew  $n^*$  observations. Now if on the first draw you received  $\bar{w}$  (the maximum wage) then clearly the next  $n^* - 1$

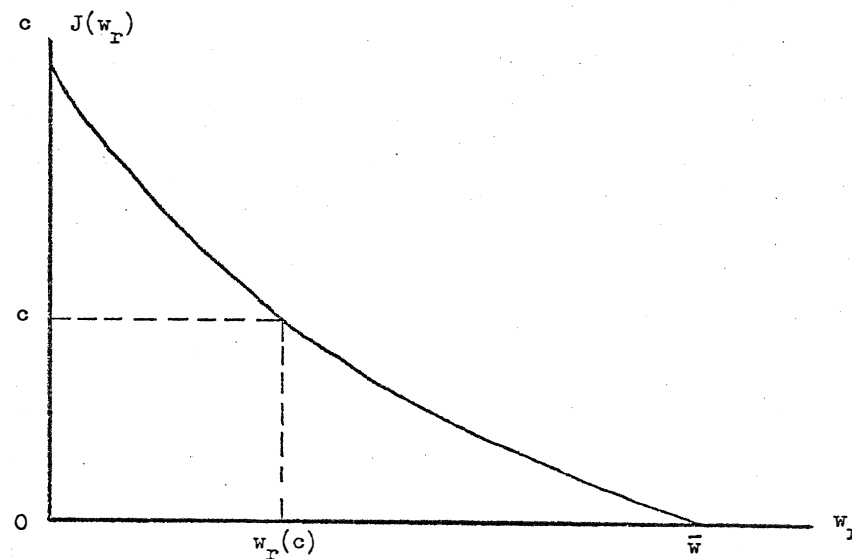


Figure 2: Reservation Wage

observations are a waste of time and resources. Using a sequential strategy you would stop after the first draw. Of course this generalizes to wage offers less than the maximum -- as soon as the worker finds a wage which beats  $w_r$  he takes it, never finding himself in a position of "going back."

As suggested above, these approaches to investment in information have a real mathematical appeal, especially the sequential rule. But this formal elegance stems primarily from the total lack of ambiguity surrounding the definition of information. Better information is always well-defined, too; it is simply a higher current wage. How information "works" is transcended, with the focus being on the technology of acquisition (or dissemination). This yields a specific, useful, formal specification of investment in information, tying costs to benefits in a way which permits integration into more sophisticated models.

The models outlined above are, of course, the simplest versions of the search problem, and many variations and extensions have been analyzed. However, they all have one major flaw which was originally pointed out by Rothschild [1973]; the distribution of wage offers,  $\phi(w)$  is taken as given. There is no explanation of where wage dispersion comes from and whether it would sustain itself over time. As Rothschild put it, "it is the theory of only one side of the market. It is a partial partial-equilibrium theory." [p. 1288]. The market models discussed below are addressed to this criticism.

But once again, I am narrowing the class of models under consideration. Following McCall's analysis, a number of papers appeared which purport to describe market behavior. These include, as a representative sample, the papers in the Phelps volume [1970], Mortensen [1970], and Lucas and Prescott [1974]. By and large, these papers fail to make explicit use of a search

rule and suffer gravely for it. Now, to be fair, I should point out that the apparent aims of these authors were not to generate pure market models which carefully integrate explicitly defined search rules. For example, the relevant authors in the Phelps volume and Mortensen were interested in explaining the Phillips Curve starting from imperfect information, and Lucas and Prescott's model is a very high-powered existence proof which yields little in the way of useful comparative statics.

Models which do introduce imperfect information in a fundamental way via some form of search activity for either the collection or dissemination of information include Mortensen [1973] Telser [1975], Butters [1975], and Wilde [1975, 1976]. Some of these models are set in the product market and some are set in the labor market, but all seek to generate a non-degenerate distribution of wages or prices endogenously. I will take the liberty of translating those which are in the product market into the labor market in order to facilitate comparisons. As mentioned in the introduction, these models can be classed according to several criteria; symmetric versus non-symmetric, sequential versus non-sequential, and according to who bears the burden of search. These notions will become clearer as we go on, but for now I remind the reader that symmetry refers to how vacancies are treated vis-a-vis unemployed workers, sequentiality refers to the search strategy, and the burden of search refers to who knocks on whose door. All these models are concerned with the existence of wage dispersion as a prerequisite to further analysis. There are clearly reasons for studying these models beyond showing that wage dispersion can exist and persist, and one cannot ignore the empirical issue of just how much wage dispersion is explained by imperfect information. But more general consideration of the effect introducing imperfect information has on market structure has awaited

resolution of the theoretical existence problem.<sup>3</sup> Fortunately, we are making some progress.

### III. MORTENSEN AND WILDE

The two papers in this section are sequential market search models. Both study conditions under which wage dispersion could arise and maintain itself over time as workers respond to it, but Mortensen is symmetric and Wilde is asymmetric. The first paper I'm going to discuss is Mortensen's. Like Telser's model this is a symmetric, sequential approach to generating a stable equilibrium which is characterized by a dispersion of wages (that is, in some sense the equilibrium is to be a distribution of wages, not a single wage as in the classical approach). Symmetry is reflected in the way vacancies are treated: exactly analogous to the way unemployed workers are treated. Let  $U$  be the number of unemployed workers in some given period and let  $V$  be the number of vacancies. Assume a constant turnover rate of  $\delta$  so that the duration of a job is  $1/\delta$ . Now if we just turn all these unemployed workers and vacancies loose and let them sample each other using some kind of sequential strategy what we have is a nightmare; we need some kind of structure on the process. Mortensen gets this structure by assuming the labor market consists of  $m$  distinct places where employers and workers can meet. At the beginning of the market period each unemployed worker goes to exactly one exchange and each vacancy is listed at exactly one exchange. These allocations of workers and vacancies are random. Since all  $m$  exchanges are identical, it suffices to consider a single representative exchange in a representative period.

Let  $x_1$  be the number of unemployed workers who arrive in the period and let  $x_2$  be the number of vacancies. The pair  $(x_1, x_2)$

is a random variable and may differ across exchanges within a market period or it may differ across market periods within a given exchange. Now, the crucial feature of Mortensen's model is how the pair  $(x_1, x_2)$  determines the wage rate which obtains on the exchange. Assume that all workers and firms are homogeneous. Using the McCall search rule, each unemployed worker sets a reservation wage,  $\underline{w}$ . Similarly, each firm (i. e. vacancy) has a reservation offer,  $\bar{w}$ , defined to be that wage rate at which the firm is indifferent to filling the vacancy or waiting and trying to hire a worker to fill the vacancy at a lower wage in the next period. Clearly,  $\bar{w} > \underline{w}$  is a necessary condition for any employment to occur, and this is shown to be true in equilibrium. Given these reservation wages (see below for explicit definitions) the actual wage is defined as

$$(5) \quad w = \begin{cases} \bar{w} & \text{if } x_2 > x_1 \\ \underline{w} & \text{if } x_2 \leq x_1 \end{cases}$$

and the number of vacancies filled is  $\min\{x_1, x_2\}$ . The logic is that within a given exchange in a given period the law of supply and demand still holds; if the supply of vacancies exceeds the demand for vacancies (unemployed workers) then the wage rises to  $\bar{w}$ . If the opposite holds then the wage falls to  $\underline{w}$ . Since the arrival of vacancies and unemployed workers is random, so is the wage. Thus, the "bargaining rule" expressed by (5) introduces the element of uncertainty needed to support wage dispersion. A constant flow of ignorance, generated by the exit of old workers and the entry of new uninformed workers, keeps the dispersion from collapsing.

Formally, this all works in the following way. If  $U$ ,  $V$  and  $m$  are assumed to be large, the  $x_1$  will be distributed via a Poisson

process with mean  $\lambda_1 = U/m$  and  $x_2$  is distributed via a Poisson process with mean  $\lambda_2 = V/m$ . Thus,  $(x_1, x_2)$  is a joint Poisson random variable with parameter  $(\lambda_1, \lambda_2)$ . Define  $F(w)$  to be the cumulative density function associated with the wage defined by (5). Then if  $\alpha(\lambda_1, \lambda_2) = \text{Prob}\{x_1 > x_2\}$ , we have

$$(6) \quad F(w) = \begin{cases} 0 & w < \underline{w} \\ \alpha(\lambda_1, \lambda_2) & \underline{w} \leq w < \bar{w} \\ 1 & \bar{w} \leq w \end{cases}$$

Finally, consider the calculation of reservation wages and offers. In spite of the fact that  $F(w)$  is discrete, treat it as continuous for the following. The reservation wage is defined by

$$(7) \quad \underline{w} + c_1 = \frac{1}{\delta} \int_{\underline{w}}^{\bar{w}} (w - \underline{w}) f(w) dw.$$

$c_1$  is the unit cost of drawing an observation (visiting an exchange).  $\underline{w}$  is included in the left hand side (costs) since it is sacrificed if the worker does not accept the job this period and samples again (that is, it is an opportunity cost). The integral gives the expected gain per period of sampling one more time, and  $1/\delta$  gives the total number of periods in which this gain will be earned.  $f(w) = F'(w)$ .

The firm is assumed to face a similar problem. Let  $\pi(w)$  be the profit associated with filling a vacancy at wage  $w$ . Then  $\bar{w}$  is determined by

$$(8) \quad \pi(\bar{w}) + c_2 = \frac{1}{\delta} \int_{\underline{w}}^{\bar{w}} [\pi(\bar{w}) - \pi(w)] f(w) dw.$$

$c_2$  is the unit cost of sampling for the firm. Otherwise the logic is exactly the same as for the worker. Using (6) in the definitions (7) and (8) yields explicit solutions for  $\underline{w}$  and  $\bar{w}$ . To get  $U$  and  $V$

endogenously (based on the entry rate of new workers), a steady-state condition, aggregate expected hires equal turnover, is utilized.  $U$  and  $V$  define  $\lambda_1$  and  $\lambda_2$  which in turn define  $\alpha$ , and the equilibrium is completely characterized.

These results, and much more, follow fairly straightforwardly. For the purposes of this survey the above will suffice, but one crucial observation is that in this symmetric specification of the market there are no externalities connected with either the firm or the worker's search efforts. Of course, this has been a very rough summary of the basic model, and Mortensen provides a very stimulating, comprehensive analysis of it.

However, flaws in a symmetric approach such as Mortensen employs center on two considerations. The first is that there is an inconsistency in the bargaining rule. Consider for a moment what is going on in the "firm" in this model. Since workers live for more than one period, any given firm will most likely have workers hired during different periods working for it. This suggests that workers with precisely the same productivity may be working side by side in the same firm but earning different wages. Any worker who is currently employed at  $\underline{w}$  could demand  $\bar{w}$  and it would pay the firm to give in to the demand; by the very definition of  $\bar{w}$  it doesn't pay to search for another worker who would work for  $\underline{w}$ . But one could apply exactly the same logic to the firm -- it could lower each worker's wage to  $\underline{w}$  and by the definition of the reservation wage it doesn't pay the worker to quit and enter the pool of unemployed. This indeterminacy (it isn't clear who would dominate, the workers or the firm) serves to point up the fact that as defined the bargaining rule is a highly suspect way to determine wages. In one sense or



another the bargaining rule must extend inside the firm. How this is to be done while preserving the symmetry is not clear.

The symmetry is problematic in another way, though; vacancies are treated analogously to unemployed workers. They are distributed randomly, each to one and only one exchange. But surely the exchange where a firm lists its vacancies will be its own employment office. In this case the number of vacancies is not random at all, it is controlled completely by the firm (what firm would allow other firms to list their vacancies in its employment office). The upshot of this and the inconsistency of the bargaining rule is that the symmetric model is just not well suited to the labor market. Mortensen's model would perhaps be better placed in the product market where repeat sales could be ignored, but even there it would be highly specialized (e.g., the housing market). And while I have discussed these problems in terms of Mortensen's model, they apply in general to symmetric models of the labor market.

So, consider the following asymmetric market search model which utilizes sequential search. It is adopted from Wilde [1975].<sup>4</sup> The fundamental problem is the same as in Mortensen's model; to study the existence of stable equilibria with wage dispersion. But the difficulties associated with the symmetric approach are assumed away at the outset. Suppose, then, that we have a labor market with homogeneous workers and firms. Let marginal value product be identical and constant for all workers. Let  $M$  workers enter the market each period, search for a job sequentially in a timeless setting, work for one period, and then exit, being replaced by an identical group which repeats the process. The simplest example is with two search cost classes of workers. Let  $B_1$  of the  $M$  workers have search cost  $c_1$  and the other  $B_2$  have search cost  $c_2$ ,

where  $0 < c_1 < c_2$ . The asymmetry is reflected in the fact that the workers do the searching. Firms act very passively, posting wage offers and simply waiting to be sampled. In equilibrium all firms must earn equal (positive) profit. Positive profits are made necessary by the linearity of the production function and the assumption that marginal value product is constant. Assume there is some fixed cost of entry which soaks up the excess. The problem is to characterize the effective supply function generated by worker search.

It can be shown quite easily that no more than two wage offers can exist in equilibrium (this is due to the profit constraint). Thus, any potential equilibrium must consist of either a single wage with all firms located at that wage, or two wages,  $w_1 < w_2$ , with  $a_1$  percent of the firms located at  $w_1$  and the other  $a_2$  percent located at  $w_2$ . Consider first the two wage equilibrium. The wage offer pair  $(w_1, w_2)$  and the split of firms across these offers,  $(a_1, a_2)$ , defines the distribution from which observations are drawn. Again let  $J(w_r)$  be the expected return to one sample drawn from this distribution when the current observation is  $w_r$ . Figure 3 illustrates the function in this case. It turns out that with a discrete distribution  $J(w_r)$  is piecewise linear, the kinks come at  $w_1$ , and it hits zero at the maximum of the wage offers. As before,  $w_r(c) = J^{-1}(c)$ .

Clearly, for a two wage equilibrium it must be the case that

$$(9) \quad w_r(c_2) \leq w_1 < w_r(c_1) \leq w_2.$$

This condition simply says that the high search cost workers will accept either wage offer but the low search cost workers hold out for the high wage offer. The question is, under what conditions does a two wage equilibrium exist. The answer is that it depends

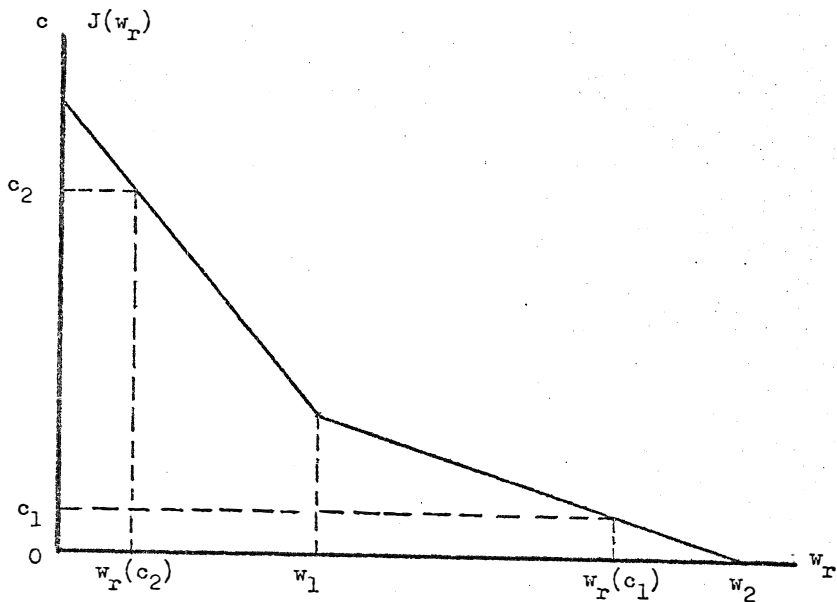


Figure 3: Two-Wage Equilibrium

on how one defines equilibrium. The most natural choice is Nash Equilibrium. Nash Equilibrium (in this setting) is defined as a set of wages ( $w_1, w_2$ ) and a distribution of firms over those wages ( $a_1, a_2$ ) such that no firm has any incentive to change its wage offer, given that all other firms maintain their equilibrium offers. A major theorem is that no Nash Equilibrium exists for the pure allocation process. Refer back to figure 3 for the two wage case. When  $c_1 > 0$  then necessarily  $w_r(c_1) < w_2$ . Thus, any firm located at  $w_2$  could lower its wage offer slightly and lose no workers, assuming all other  $w_2$  firms don't change their wage offers so that the expected return to search function stays the same. But lowering the wage offer without losing any workers means that profits increase beyond  $\pi$ , breaking the potential Nash Equilibrium.

There is no single wage Nash Equilibrium either. Suppose all firms were offering some wage  $w_0$  which just allows the profit constraint to be satisfied. Even though the worker expects all wage offers to be  $w_0$ , if he faces positive search costs he will set a reservation wage strictly less than  $w_0$ . This implies that if all other firms stay at  $w_0$  then any single firm could get away with lowering its offer ever so slightly without losing any workers. Again, this breaks the potential single wage Nash Equilibrium. At the same time, it suggests that the model needs a little more structure; the same argument which shows the single wage equilibrium cannot be Nash implies that the wage would eventually fall to zero. Therefore, we need to introduce some element of worker supply. Suppose there is some underlying supply curve; a distribution of limit wages below which workers will not take jobs regardless of alternatives in the market. In this case Nash Equilibria may exist, but the only non-pathological one is degenerate at the monoposony wage (the wage which would obtain if a single employer controlled the market and could extract all economic rent from the workers).

While this replicates a result obtained by Diamond [1971] in a more dynamic setting, it is not a very satisfying state of affairs. It would appear that sequential search is inherently unstable, and the slight degree of monopsony power it accords each firm tends to mushroom, driving the wage down to its lower limit.

I have extended the simple model outlined above in several directions, all of which exhibit a fundamental instability. When limit wages are introduced the equilibrium is dominated by assumptions made concerning the distribution of the limit wages, and search, *per se*, takes a back seat. Part of the problem is that workers never directly compare wage offers, they always compare the current offer to some mythical reservation wage. If direct wage competition of some kind (not necessarily complete or perfect competition) is introduced, then a stabilizing spread of offers may take place. This is the essence of non-sequential sampling. Before getting into that, though, it is worth noting that the definition of equilibrium was crucial to the above conclusions concerning asymmetric, sequential models. It can be shown that entire families of a weaker type of equilibrium exist in the pure allocation model (that is, independent of limit wages). The equilibrium is similar to those used in other recent works in the economics of information (for example, see Spence's [1974] analysis of market signaling), but I have argued that these equilibria are inappropriate and that the Nash Equilibrium is the relevant one. But rather than dwell on these subtleties let us turn to the asymmetric, non-sequential models.

#### IV. BUTTERS AND WILDE

The difficulties with sequential approaches to the market search problem are manifest. One way out of the bind (inconsistency

or degeneracy) is to introduce alternate kinds of search strategies. Two models which do so are Butters [1975] and Wilde [1976]. Butters places the burden of search on the firm and Wilde places it on the worker. The resulting models both generate stable, non-degenerate wage distributions which are surprisingly similar. But there are some surprising differences, too. In particular, it turns out that when the burden of search rests with the firm there are no externalities present, but when the burden of search is imposed on the worker there are both positive and negative externalities associated with search activity. Let us turn to the unique and elegant example of an asymmetric non-sequential market search model provided by Butters' model of advertising in the product market. The relevance of an advertising model to the current survey is that Butters treats advertising as a form of seller search. When translated into the labor market this yields a model in which the burden of search falls on the firm. The simplest version of Butters' model is a one-period, static analysis in which each buyer in the market is interested in purchasing a single unit of some good. The buyer's strategy is simple; he just collects ads (*i. e.*, waits to be sampled) and after all ads are in he selects the one offering the lowest price (highest wage).

The driving force of the model lies on the firm side. The good is assumed to be produced at constant costs (analogous to constant marginal value product) and advertisements are sent out randomly at some constant cost per ad, also. The key is the profit constraint -- it is used to characterize the properties of any potential equilibrium distribution of prices and sales (wages and hires). The number of ads sent out is traded off against the effectiveness of ads at a given price to maintain zero profits

across the entire range of prices offered in equilibrium. The interested reader can refer to Butters for the details of this derivation. The main results, though, are that if  $g(p)$  is defined as the density of ads per buyer at price  $p$  and  $a(p)$  is defined as the density of sales at price  $p$ , then

$$(10) \quad g(p) = \begin{cases} 0 & p < p_0 + b \\ 1/(p - p_0) & p_0 + b \leq p \leq m \\ 0 & m < p \end{cases}$$

$$(11) \quad a(p) = \begin{cases} 0 & p < p_0 + b \\ b/(p - p_0)^2 & p_0 + b \leq p \leq m \\ 0 & m < p \end{cases}$$

These are the equilibrium distributions.  $p_0$  is the per unit production cost,  $m$  is a limit price above which buyers refuse to purchase the good, and  $b$  is the unit cost of an advertisement. Thus,  $p_0 + b$  is the "competitive price" in this market (the minimum price which just covers the cost of producing the good and informing the consumer about it) and  $m$  is the "monopoly price." The equilibrium is a Nash Equilibrium in which prices spread out between the competitive price and the monopoly price. Stability derives from the ability of buyers to directly compare prices. This, in turn, derives from the non-sequential search strategy which firms use -- the buyer is able to wait until all ads are in before making a decision.

A final comment: just as in the symmetric, sequential model (Mortensen), there are no externalities in the above model. The search activity of any firm confers no positive or negative benefits on other firms which are not captured in the profit function. As I will show in a moment, this fails to hold up when the burden of search rests with the workers.<sup>5</sup>

Consider Wilde [1976]. This is another non-sequential model. The basic structure is the same as the last two models discussed in this survey. Firms and workers are homogeneous, marginal value product is constant, and a profit constraint is imposed on the firm side of the market. The difference is in the search strategy. Note that in Butters' model the firms actually used a strategy very similar to Stigler's optimal sample size strategy, discussed in the early sections of this paper. Suppose we employ this technique (Stigler's) as a non-sequential strategy for the workers in a market setting, instead of using a sequential strategy. Then since all workers are identical they will choose the same optimal number of samples to draw. Let this be  $n^*$ . Now just as in Butters' case, the only candidates for equilibrium are distributions of wages which spread out over some interval, say  $[\underline{w}, \overline{w}]$ . Let  $F(w)$  be some such candidate. I claim that no  $F(w)$  can simultaneously satisfy the definition of a distribution function and satisfy an equal profit constraint for all wage offers in the interval  $[\underline{w}, \overline{w}]$ . To see this, the first thing to note is that there must be a mass point located at  $\underline{w}$  (i. e.,  $F(\underline{w}) > 0$ ). If this were not the case then, assuming that  $n^* > 1$  (the only interesting case), no firm at  $\underline{w}$  could ever attract any workers; if workers draw more than one observation and there is no mass point then the probability of drawing all observations at  $\underline{w}$  is zero. But once the atom is admitted at  $\underline{w}$  the model becomes inconsistent; the mass point introduces a discontinuity which cannot be adjusted for. This problem arises because the only way a firm which is offering  $\underline{w}$  attracts a worker is if that worker draws all his observations from firms offering  $\underline{w}$ . The existence of a mass point at  $\underline{w}$  makes this possible, but if a worker does draw all  $n^*$  offers from the mass point, then he is indifferent to which firm he works for and must choose one of the firms at random. If any

of the firms located at  $\underline{w}$  were to raise its offer by a very small amount, it would break the tie and the expected supply of workers would jump. Translating this into profits introduces the discontinuity. So, in the simple optimal sample size model there is no non-degenerate equilibrium. There is no single wage equilibrium either unless limit wages are introduced, but once again the Nash Equilibrium is at the monopsony wage.

The optimal sample size model is close; it does allow for a comparison of wage offers which spreads the distribution, but the ability of a worker to guarantee himself multiple observation unravels the equilibrium from below. The question is how to modify the optimal sample size model to maintain the spread of offers but eliminate the discontinuity while keeping the burden of search on the worker. The answer is to make some workers extra unlucky in the sense that they not only draw  $\underline{w}$  as an offer, but that is the only offer they draw. The idea is to make the actual number of observations yielded by a particular investment in search a random variable. The specific assumption used is that the number of observations is a poisson process with mean  $\lambda$ . Search costs are assessed in proportion to  $\lambda$ , which is the decision variable of the searcher.

Formally, let  $F(w)$  be some distribution of wage offers. Then if  $n$ , the number of actual observations drawn from  $F(w)$ , is poisson with mean  $\lambda$ , it can be shown that the c.d.f. of the maximum wage offer, given  $\lambda$ , is

$$(12) \quad H(w;\lambda) = \exp \{ - \lambda [ 1 - F(w) ] \}$$

Let  $h(w;\lambda) = \partial H(w;\lambda) / \partial w$ . Then if  $F(w)$  is defined on  $[\underline{w}, \bar{w}]$ , the workers problem is to pick that  $\lambda$  which maximizes expected net income.

That is,

$$(13) \quad I^* = \max_{\lambda} I(\lambda) = \max_{\lambda} \int_{\underline{w}}^{\bar{w}} wh(w;\lambda) dw - c\lambda,$$

where  $c$  is the cost of search, assessed proportionally to  $\lambda$ , the intensity of search.

The above arguments justified this formalization of the technology of search from an analytical point of view. Does this treatment have any economic content? After all, it was argued that sequential search dominates non-sequential strategies. The basic non-sequential element can be justified on cost grounds; there are lags and structural factors present in the labor market which make it more efficient to gather a batch of observations, check the highest offer, and decide based on it whether to accept a job or gather another batch of observations. The element of uncertainty is justifiable directly; sometimes a searcher is lucky and gets lots of observations for his efforts and other times he gets on the wrong bus, spends time following up on false leads, or doing any one of a number of unproductive things. Thus, it appears that the uncertain non-sequential search rule is reasonable.

To close the model introduce the firm side. Assume some equilibrium  $F(w)$  exists and all workers are searching at intensity  $\lambda^*$ . Manipulating the poisson distribution, it can be shown that the effective supply function these generate is

$$(14) \quad S(w) = \frac{M}{N} \lambda^* \exp \{ - \lambda^* [ 1 - F(w) ] \}$$

where  $M$  is the number of workers and  $N$  is the number of firms. Putting this supply function into the profit constraint allows one to solve for  $F(w)$  in terms of  $\lambda^*$ . Using this expression for  $F(w)$  in

(12) and (13) defines  $I(\lambda)$  independent of  $F(w)$ ; that is,  $I(\lambda)$  is defined totally as a function of  $\lambda$  and the various parameters of the model. Taking the derivative of  $I(\lambda)$  and setting it equal to zero yields the optimal  $\lambda^*$ . This, in turn, characterises the equilibrium  $F(w)$  along with its limits,  $\underline{w}$  and  $\bar{w}$ , which are now endogenous variables. This is essentially the same process used by Butters. However, a special problem arises concerning the limit wage,  $w_L$ , which must also be introduced here to control the lower limit of the distribution of firms.

As derived above, the equilibrium is not stable. Consider firms at the lower limit,  $\underline{w}$ . Since it can be proved that  $F(w)$  cannot have mass points, any firm which offers  $\underline{w}$  could lower its offer and lose no one; it is already being beat by everyone, so offering a lower wage cannot hurt (assuming all other firms stay where they are). It must be the case that  $\underline{w} = w_L$ . But  $\underline{w}$  is endogenous. That is, unlike Butters' model, when the intensity of search decision is placed on the side of the market which does not suffer the profit constraint, we cannot set an arbitrary  $w_L$  and let the distribution adjust to it. Breaking the profit maximization decision away from the intensity of search decision sacrifices a degree of freedom.  $\underline{w}$  (the lower limit of the distribution of wage offers) becomes endogenous and may or may not equal some exogenously set  $w_L$ . The upshot of this is that an additional constraint is imposed on the model,  $\underline{w} = w_L$ . Satisfying this constraint requires that the model be opened up in some way; something previously taken as fixed must become endogenous. I consider two cases; the first takes total search-related costs as fixed and lets the mix across firms and workers be endogenous. The second possibility is to let the number of firms be endogenous. Surprisingly, the two versions are very much

alike. In particular, the equilibrium distribution of wage offers in either case is

$$(15) \quad f(w) = \left[ \frac{1}{\lambda^*} \right] \left[ \frac{1}{\hat{w} - w} \right]$$

where  $\lambda^*$  is the optimal intensity of search, a function of various parameters which depend on which model one considers and  $\hat{w}$  is marginal value product. Compare this to (10).  $f(w)$  is defined over  $[w_L, w_c]$  where  $w_c$  is the "competitive" wage.  $w_c$  is sensitive to various parameters; since  $f(w)$  must integrate to 1, anything which effects  $\lambda^*$  must also effect  $w_c$ . In any case, the interested reader can refer to the full paper to see how the details of the model work, what is important here is that there are both positive and negative externalities present in this setting.

Where do these externalities arise from? Consider the equilibrium wage offer distribution (15). Clearly, the distribution is sensitive to  $\lambda^*$ . But  $\lambda^*$  is the common equilibrium intensity of search. The point to note is that each individual worker's search efforts contribute to the aggregate and generate effects which work in the same direction as aggregate shifts in  $\lambda^*$ . Thus, an increase in any individual worker's search intensity causes  $f(w)$  to shift down. But this implies  $w_c$  must increase. This increases the spread of wage offers in favor of higher wages, generating a positive externality as this shift toward higher wage offers increases the return to search for other workers. However, stability requires that the spread of wages continue to extend down to  $w_L$ . The increased intensity of search on the part of the individual worker makes it harder for firms located near  $w_L$  to cover their costs (either fixed or search related). The burden of these costs is shifted to other workers, either directly or via the

number of firms in the market. Regardless of how these externalities are channeled, the point remains that there is no reason to expect an optimal investment in search.

## V. CONCLUSION

The comments in this paper may seem vague to some and not rigorous enough to others. Unfortunately the models are somewhat mathematical and do not lend themselves to being surveyed, but the points I have made suggest the following. Market search models can be categorized according to who bears the burden of search (this subsumes symmetry) and whether search is sequential or non-sequential. The original question that was posed, under what conditions will costly information generate and support non-degenerate price or wage dispersion, has faded into the background in the light of recent research. The problems which face us now are centered around the selection of a sensible equilibrium notion and the problem of efficiency. Comparing Butter's model with Wilde suggests that the efficiency problem is closely related to the issue of who bears the (physical) burden of search. This last question is a fascinating, and important problem which has never been addressed directly.

## FOOTNOTES

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1. Taking advantage of my position as surveyor, I have chosen not to outline justifications for studying markets with imperfect information in the first place. This has been done elsewhere (see, for example, the excellent and now classic survey by Rothschild [1973]), and since I am surveying only a very small subset of the search literature, I think that the discussion does not bear repetition.
  2. The reservation wage is that wage offer at which the worker is indifferent to accepting the offer (ceasing search efforts) and continuing to search for a higher wage. It is similar to a limit wage but generally defined endogenously. See equation (4) for a formal definition.
  3. Once again I refer the interested reader to Rothschild [1973] for a further discussion of these issues.
  4. Recently I have been informed of some related work conducted at the Federal Reserve Board. See Salop and Stiglitz [1975] and

von zur Muehlen [1976]. These models are set in the product market but have much in common with my own analysis.

5. While my discussion has centered on labor market models, I have left Butters in its product market form in order to retain the spirit of advertising. Modification into a labor market setting is straightforward given Butters' assumptions.



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